Effects of symmetry breaking on statistical properties of near-lying acoustic resonances

David M. Leitner
Department of Chemistry, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801
(Received 26 June 1997)

Statistical properties of acoustic modes in quartz blocks, measured recently by Ellegaard *et al.* [Phys. Rev. Lett. **77**, 4918 (1996)], are compared with predictions of a random matrix ensemble that incorporates the approximate point-group symmetry of the samples. We observe close agreement between the nearest-neighbor acoustic mode spacing distributions and level spacing distributions predicted for the corresponding matrix ensemble. The single symmetry-breaking parameter of the random matrix ensemble is seen to scale with the size of the symmetry-breaking deformation of the quartz sample. [S1063-651X(97)13110-5]

PACS number(s): 62.30.+d, 02.20.Df

Statistical properties of near-lying acoustic modes, such as the distribution of spacings between mode frequencies, have been shown to be well described by the Gaussian orthogonal ensemble (GOE) [1,2]. Recently, however, Ellegaard *et al.* [3] demonstrated that acoustic mode statistics differ from GOE predictions for quartz blocks whose point-group symmetry is broken. This development is reminiscent of the early studies of energy-level statistics of complicated quantum-mechanical systems. Several decades ago, Rosenzweig and Porter observed, upon analyzing level spacing distributions for families of transition-metal atoms, that symmetry breaking due to spin-orbit coupling leads to deviations from GOE predictions [4]. Random matrix ensembles modeling quantum systems with broken symmetry have since been the focus of a number of studies [4–8]. While the spectral statistics predicted by these matrix ensembles agree well with results of numerical studies of model quantum systems [9], comparisons with experiment have been limited to rather small numbers of levels [10]. Ellegaard *et al.*'s data [3] provide, for the first time, a sizable set of experimental levels with which to compare predictions of random matrix models that incorporate point-group symmetry breaking. It is important to remark that the quartz blocks more closely resemble scalar pseudointegrable systems than chaotic ones [3], which limits the range of levels whose statistical properties would be shared by those of the matrix ensembles. The acoustic modes of the quartz blocks are expected to be statistically indistinguishable from energy levels of quantum ''chaotic'' systems with broken symmetry over distances of a few mean level spacings [3].

The quartz samples analyzed in Ref. [3] possess an approximate twofold ''flip'' symmetry about one axis. Flip symmetry of the quartz block was broken by removing a small piece of radius r from the sample. If $r=0$, due to flip symmetry, statistics of near-lying eigenmodes correspond to those of two randomly superposed sequences of GOE levels. Sufficiently large r breaks the symmetry so that acoustic mode statistics resemble those of the GOE. For smaller r, Ellegaard *et al.* showed that the eigenmode statistics were of intermediate character, and proposed a random matrix ensemble to model the quartz samples [3]. That ensemble, here called $E(\Lambda)$, where Λ is the symmetry-breaking parameter, was analyzed in earlier studies [5,7,8]. Ref. [3], however, provided no comparison between $E(\Lambda)$ and the acoustic statistics of the quartz blocks, nor any connection between Λ and r. Here we show that the nearest-neighbor eigenmode spacing statistics of the samples follow predictions of $E(\Lambda)$ quite well by comparing results reported in Ref. [3] with an analytical prediction for $E(\Lambda)$ [8], and thereby observe a relation between Λ and r.

The random matrix ensemble $E(\Lambda)$ consists of matrices $H=H_0+V$, where H_0 is block-diagonal, here two blocks for the two-fold symmetry, each a member of the GOE; V couples the blocks, breaking the symmetry. Level statistics of $E(\Lambda)$ vary with $\Lambda=\epsilon^2\rho^2$, where ϵ^2 is the variance of the random elements of V, and ρ is the density of eigenmodes. Equation (10) of Ref. [8] provides a very close approximation to the nearest-neighbor level spacing distribution $P_S(S;\Lambda)$ of $E(\Lambda)$. For the quartz samples, where H_0 consists of two blocks of the same dimension, the level spacing distribution reads

$$P_S(S;\Lambda)=c_N c_D\left(\sqrt{\pi/32\Lambda}\,I_0\left[\frac{c_D^2S^2}{16\Lambda}\right]Se^{-(c_D^2S^2/16\Lambda)(1+2\pi\Lambda)}+\frac{\pi}{8}Se^{-(\pi/16)c_D^2S^2}\mathrm{erfc}\left[\frac{\sqrt{\pi}}{4}c_DS\right]\right), \tag{1}$$

where I_0 is a Bessel function, and c_D and c_N are set so that $\langle S\rangle=1$ and P_S is normalized, respectively. Equation (1) fits numerical results of $P_S(S;\Lambda)$ for the ensemble $E(\Lambda)$ defined above closely over the range $\Lambda=0$ to $\Lambda\approx0.35$ [8], at which $P_S(S;\Lambda)$ is already difficult to distinguish from $P_S(S)$ in the GOE limit. In Ref. [3], nearest-neighbor eigenmode spacing distributions were plotted for several values of r. Figure 1 compares a fit of Eq. (1) to three distributions shown in Fig. 2 of Ref. [3]. The distributions for the acoustic mode spacings are seen to compare well with those for $E(\Lambda)$, with $P_S(S;\Lambda)$ from the experimental data closely following the predicted distributions; only for the smaller Λ is the maximum in $P_S(S;\Lambda)$ apparently slightly higher than predicted. For the distributions plotted in Ref. [3], corresponding to $r=0$, 0.5, 0.8, 1.1 and 1.4 mm, we find Λ to be 0.0013, 0.0054, 0.0096, 0.0313, and 0.0720, respectively. The $r=$ 1.7 mm distribution is also plotted in Fig. 2 of Ref. [3]; this distribution appears similar to that plotted for a sample with

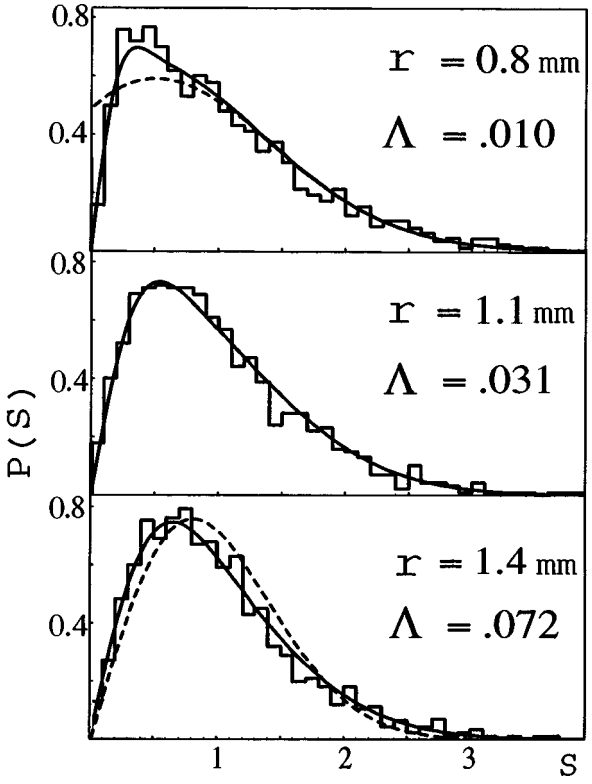

FIG. 1. Fit of Eq. (1) to three eigenmode spacing distributions from Ref. [3] (histograms). Dashed curves in the top and bottom plots result from, respectively, two superposed independent GOE sequences and one sequence, the former corresponding to conserved symmetry.

no apparent twofold symmetry. The latter two distributions fit to $\Lambda=0.113$ and 0.138, respectively.

We now consider how the experimental symmetry breaking parameter r and the matrix ensemble parameter Λ are related. Since the acoustic resonance density ρ is largely independent of small r, Λ varies with r through ϵ. Mode coupling occurs at the surface of the quartz block, and we thus expect $\epsilon\sim r^2$, or $\Lambda\sim r^4$. The data reflect this scaling. Neglecting the $r=1.7$ mm distribution, already close to the GOE limit as noted above, we see that the distributions for the other r fit to $\Lambda=a+br^4$, with $a=0.003$ and $b=0.018$ mm^{-4}, which is shown in Fig. 2. That $a\neq 0$ reflects slight differences between statistics of two superposed GOE levels, and the mode spacing distribution for $r=0$ plotted in Ref. [3]. Using this fit for Λ, we can estimate the coupling ϵ between $r=0$ modes; e.g., since $\rho\approx 4.7$ kHz^{-1} [3], $\epsilon\approx 0.03$ kHz when $r=1$ mm. Still lacking is a more complete theoretical relation between $E(\Lambda)$ and the samples, as obtained for quantum systems with approximate point-group symmetries [9].

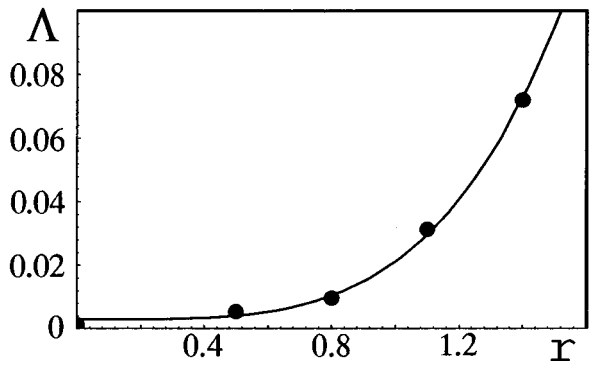

FIG. 2. Points are values of the symmetry-breaking parameter Λ of the random matrix ensemble, obtained by fitting Eq. (1) to the distributions of nearest-neighbor acoustic mode spacings reported in Ref. [3] for several values of the experimental parameter, r. The curve is a best fit of Λ using the form $\Lambda=a+br^4$ ($a=0.003$ and $b=0.018$ mm^{-4}).

Comments from Professor R. L. Weaver and support from NSF Grant No. CHE 95-30680 under the direction of Professor P. G. Wolynes are gratefully acknowledged. The author thanks the Department of Chemistry, Bilkent University, Ankara, Turkey, for its hospitality during a visit when part of this work was done.

[1] R. L. Weaver, J. Acoust. Soc. Am. **85**, 1005 (1989).
[2] C. Ellegaard, T. Guhr, K. Lindemann, H. Q. Lorensen, J. Nygård, and M. Oxborrow, Phys. Rev. Lett. **75**, 1546 (1995), and references therein.
[3] C. Ellegaard, T. Guhr, K. Lindemann, J. Nygård, and M. Oxborrow, Phys. Rev. Lett. **77**, 4918 (1996).
[4] N. Rosenzweig and C. E. Porter, Phys. Rev. Lett. **120**, 1698 (1960).
[5] F. J. Dyson, J. Math. Phys. **3**, 1191 (1962).
[6] J. B. French, V. K. B. Kota, A. Pandey and S. Tomsovic, Ann. Phys. (N.Y.) **181**, 198 (1988).
[7] T. Guhr and H. A. Weidenmüller, Ann. Phys. (N.Y.) **199**, 412 (1990).
[8] D. M. Leitner, Phys. Rev. E **48**, 2536 (1993).
[9] D. M. Leitner, H. Köppel, and L. S. Cederbaum, Phys. Rev. Lett. **73**, 2970 (1994).
[10] See Ref. [7] for one such comparison with the available levels of ^{26}Al.